The interpretation of $\varkappa$ as a transformation parameter according to Eq. (20) allows some additional information on $\varkappa$ which is not readily obtained from the Schrödinger equation:

a) In Sect. 4.2 we introduced the quasi spin quantum number c in order to generate the actual PSE. As seen from Fig. 6, by rotating the hydrogen spectrum onto the PSE spectrum, certain states (indicated by open dots in the figure) become forbidden. However, if one postulates that no states are to be lost by the transformation, then one is forced to infer the existence of an additional level splitting effect such that the total number of allowed states remains constant.

b) From the Schrödinger treatment the only condition on $\varkappa$ is $\varkappa > 0$. If Eq. (20) is assumed to be indeed correct, than the allowed range for $\varkappa$ is $0 < \varkappa \leqq 1$. Still, neither by Eq. (6) need $\lambda(\varkappa)$ to be an integer in order to define a solution of the Schrödinger Eq. (3) nor is δ by Fig. 6 necessarily confined to special angles. However, from Fig. 6 it is clear that for intermediate angles $\pi/3 > \delta > 0$ the spectra do not coincide with lattice points and one has no pure states. If one considers only pure states, then λ assumes indeed only integer or even integer numbers.

c) The order of filling of the states is obtained from Fig. 6 by assuming a small perturbation which tilts the spectra by an infinitesimal angle. If the hydrogen spectrum is tilted just a bit toward the PSE-spectrum, the order of filling according to increasing λ is correctly inferred as $(n, l) = 1s;\ 2s,\ 2p;\ 3s,\ 3p,\ 3d; \ldots$. Tilting the PSE-spectrum a bit towards the hydrogen spectrum, the order of filling is correctly deduced to be $(\bar{n}, l) = 1s;\ 2p,\ 2s;\ 3d,\ 3p,\ 3s;\ 4f, \ldots$.

5. Final Remarks

By using the existence of a freely adjustable parameter $\varkappa$ in the simple non relativistic Schrödinger equation for the Coulomb problem, we interpreted the occurance of only double weighted angular momentum quantum numbers in the conventional classification of the PSE (Fig. 1) as being due to a special choice of $\varkappa$. It seems that the most degenerate representation of the PSE requires the filling of only four Coulomb shells with a fourfold degeneracy of the states as compared with the twofold degeneracy due to spin in the conventional classification of the hydrogen spectrum. In the frame of the present simple theory, $\varkappa$ appears to be a rather formal parameter. The fact that this parameter proves to be useful in describing the transformation between the PSE and the hydrogen spectrum suggests, that a more fundamental interpretation should be possible within the scope of a more general treatment of the Coulomb problem. Similarly, the existence of an additional level splitting effect requires a more detailed theory for its explanation then used in this communication. It has been shown recently [6] that the H-atoms can be described by the dynamical group $0(4,2)$ in a 6-dimensional space. This group is the product $0(4,2) = 0(2,1) \times 0(2,1)$ of two independent Lorentz groups and the effect of spin in one of them should require by reason of symmetry the same in the other; the fourfold degeneracy of the states in the PSE appears then to be quite natural.

The high symmetry of the PSE seems to be of interest from different point of views and further questions arise immediately as may be indicated by the following remarks.

1. Starting from a PSE with only four Coulomb shells, detailed calculations in atomic and solid physics require a quite unusual first order approximation since in contrast to current theories the states with high orbital angular momenta in a shell have – similar to the nuclear states – a lower energy than those with low orbital angular momenta. Further, the tetrahedral bond which is very characteristic for many systems, might perhaps find a more direct interpretation than in terms of an sp^3 hybrid bond.

2. Taking the isotopic spin into account, the nucleus is described basically by five quantum numbers. One expects these quantum numbers to match somehow the five quantum numbers of the electron shell if the total wave function of a complete atom consisting of nucleus and shell is considered. What are then the properties of nuclei corresponding to the atomic magic numbers $Z = 4,\ 20,\ 56,\ 120$? Since for the nucleus a harmonic oscillator potential is a reasonable approximation, it is interesting to note that the total number of states of a three dimensional harmonic oscillator is for every second shell identical to an atomic magic number. Unfortunately, this

[6] A. O. Barut and H. Kleinert, Phys. Rev. **161**, 1464 [1967]. – Y. Ne'eman, Algebraic Theory of Particle Physics, Benjamin 1967.