In Fig. 3 the electronegativity [3] of the elements is plotted for each chemical group as a function of $N=n+l$. The elements with odd N belong to the lower halves of the double shells and are represented by black symbols, the elements with even N are in the upper halves and are represented by open symbols. Two consecutive elements of the same group are connected by a full line if they belong to the same double shell, and by a dotted line if they belong to different ones.

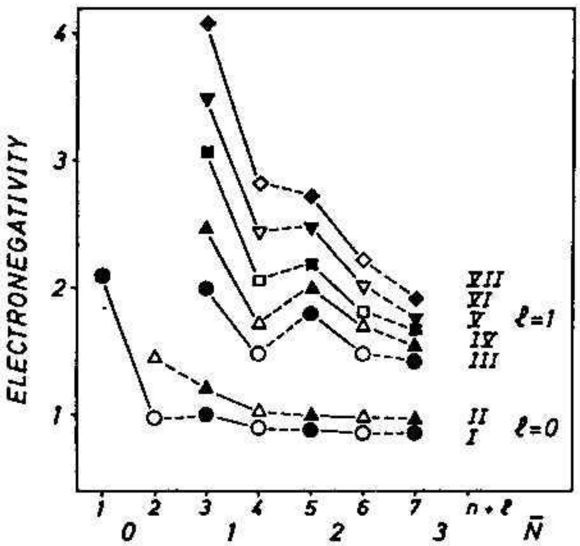

Fig. 3. Double shell structure in the electronegativity along the chemical groups.

The double shell structure is very pronounced in the group III elements (B, Al, Ga, ...). Clearly, the slope of the first dotted line is here opposite to the slope of the full lines. However, due to the strong background variation, the opposite slopes of full and dotted lines are in many cases suppressed; this fact should not give rise to the impression that two elements of different double shells are *basically* more similar than two of the same double shell.

Similar curves may be obtained for other properties which depend on the outer electrons. As a rather basic property the ionization energies [4] are shown in Fig. 4. Opposite slopes are barely indicated, but the different general appearance of the full lines compared to the dotted lines is in accordance with the assumption of double shells in the vertical direction. One can show by appropriate plots that with increasing nuclear charge of isoelectronic atoms the double shell structure in the ionization energies for the last electron becomes less pronounced in accordance with the observation that highly ionized atoms obey [2] the hydrogen level spectrum, which, of course, does not have a vertical structure.

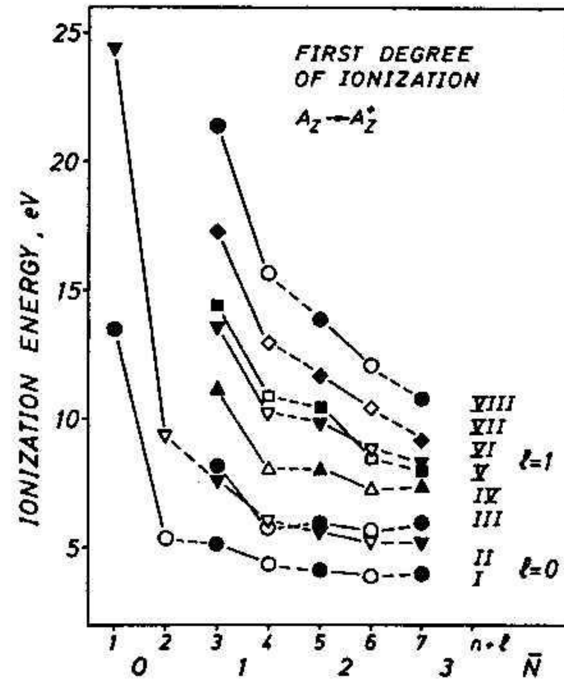

Fig. 4. Double shell structure in the ionization-energies along the chemical groups.

4. Theoretical Interpretation

4.1. Solutions of the Schrödinger equation

In solving the Schrödinger equation for a Coulomb potential, there is some freedom in the choice of a certain constant $\varkappa$. This fact is well known [5], but in the past only that value of $\varkappa$ which leads to the spectrum of the H-atom has been considered to be of significance. In the following the Coulomb problem shall be reconsidered by paying special attention to the role of $\varkappa$ with respect to the generation of different spectra.

In spherical coordinates, the radial part of the wave equation is given [5] by:

$$-\frac{\hbar^2}{2\mu}\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}R}{\mathrm{d}r}\right)-\frac{Z e^2}{r}R+\frac{l(l+1)\,\hbar^2}{2\mu r^2}R=E\,R\,. \tag{1}$$

We consider only bound states, i. e. the energy is assumed to be negativ, $E=-|E|$. We define the dimensionless quantities ϱ, $\varkappa$, λ by

$$\varrho = k\,r/\varkappa\,, \tag{2a}$$

$$1 = 2\,\mu\,|E|/\hbar^2 k^2, \tag{2b}$$

$$\lambda = \varkappa\, 2\,\mu\, Z\, e^2/\hbar^2 k \tag{2c}$$

and obtain Eq. (1) in the dimensionless form

$$\frac{\mathrm{d}}{\mathrm{d}\varrho}\left(\varrho^2\frac{\mathrm{d}R}{\mathrm{d}\varrho}\right)-[l(l+1)-\lambda\,\varrho+\varkappa^2\varrho^2]\,R=0\,. \tag{3}$$

[4] Lit. [3], p. 20.

[5] L. Schiff, Quantummechanics, McGraw-Hill 1955, p. 82.